Wavelet Based Fractal Analysis: An alternative approach for facies classification
Juan R. Jiménez*, Rafael Banchs, y Reinaldo J. Michelen, PDVSA-Intevep; Adrian Peinado, Universidad Simón Bolívar

abstract
Wavelet transforms (WT) have become a novel and powerful signal analysis tool. Its applications, that range from medical to engineering, have succeed in simplifying the handling and processing of a wide variety of signals.

In this paper we explore some of the possibilities to perform this analysis, starting from an application of WT in biomedical engineering: wavelet based fractal analysis (WBFA), introduced by Akay (1997) that seems to be applicable to the analysis of seismic and well log information and then elaborating on some possible extensions of this application.

To explore the applications of WBFA in reservoir characterization, we set up a series of experiments on a very simple geological scenario described by two wells that show an increment of the shale content in the reservoir in a preferential direction. We applied the WBFA to the well logs and the seismic information related to these two wells and we were able to show that the WBFA have the potential to conform a facies description system that can be used to map facies changes at both well logs and the seismic level.

Introduction
When we explore issues related to lithological or facies classification, one general problem always arises. Given the set of properties measurements available at the reservoir area and level what is the optimum classification system that can be devised using this information and what is the prediction power of such a system for the classification of additional sites not incorporated in the original definition of the system.

At the well log scale, a popular scheme of representation have been the use of cross-plots between well logs, with the density-neutron cross-plot, being probably the most representative of these schemes for the separation between lithologic units.

Sometimes, when a full set of well logs have been recorded for a particular area well log information can be optimally used by defining the axis of the cross-plots as linear combination of the available well logs using the well known statistical technique of Factor Analysis. These factors are then the linear combination of the well log values. To classify any new site, i.e. a proposed well location, the factor value for that well have to be calculated provided that the exact same set of logs have been recorded for the new well.

The relevance of the seismic information in these type of classification scheme become evident when we want to characterize a new target, that is a subsurface location that have not been drilled. The general procedure is then to associate the reservoir facies - previously characterized at well log resolution at drilled locations - to their seismic signature, which is in turn described by the seismic parameters or attributes. The seismic signature of the new target is computed and then classified relative to the end members defined by the drilled locations. If a consistent scheme - one that, at least passes the cross-validation test - can be generated using this approach then every trace of the seismic survey enclosing the well locations can be classified in terms of the defined end members.

With the introduction of wavelet transforms (WT) and its advantages to the Fourier methods in signal analysis the obvious quest for applications into the seismic world started with the pioneer work of Jean Morlet in the early eighties and the subsequent development by (?) (?) and many others. The question here is then whether a system of classification can be generated using the wavelet transforms to characterize the logs and seismic signals instead of the traditional seismic attribute approach.

We show in this paper a method borrowed from the biomedical literature (?), wavelet based fractal analysis, that can be used to analyze the information contained in the wavelet coefficients. We first test the method with resistivity logs recorded in two wells that penetrate shally and sandy environments respectively, and obtain the variation
of the variance of the coefficients with scale shows
distinctive behaviors depending of the dominant
lithology around the each well. Then, we repeat
the process with acoustic impedance logs from the
same pair of wells and 3D seismic data that cov-
ered a larger area, and obtain responses that can
be used also for facies classification.

Wavelet Based Fractal Analysis (WBFA)
The wavelet transform is a very useful tool in the
analysis of nonstationary signals due to their abil-
ity to resolve features at various scales. In par-
cular, on the of the most promising application
has been the analysis of the variance of a physical
process across different scales.

In a interesting and novel approach Akay (1997)
used the technique of Wavelet Based Fractal Anal-
ysis (WBFA) for the computation of the fractal
dimension of heart-sound waveforms. Akay’s anal-
ysis consisted in the calculation of the variance of
the wavelets coefficients (the detailed signals) and
plotted versus scale on a log-log plot. At each scale,
the detailed signals are assumed to be stationary.
Regions of linearity in this kind of plot correspond
to a power-law process over a particular region of
frequencies, with the exponent of the power-law
process being related to the slope of the line (\(\beta\)).

FIG. 1. WBFA (bottom figure) of heart-sound
waveforms of normal human subject (top figure) and a
subject with multiple coronary occlusions (middle fig-
ure). The analysis was performed on the quietest part
of the signal, contained between the arrows. (taken
from Akay [1997])

Figure 1 shows the original Akay’s analysis per-
formed on heart-sound waveforms showing the ef-
fact of the coronary artery disease. The normal
heart-sound waveform contain less high frequency
energy than the abnormal one. WBFA performed
on both signals reveals larger variances at all scales
for the abnormal heart-waveform. Moreover, the
variation of the variance with scale follows differ-
ent power-laws for each case which, according to
Akay, suggests the possibility of devising electronic
instruments capable of helping physicians detect
coronary ischemia in its early stage. Akay’s re-
results suggests also the possibility of using WBFA
to classify geophysical signals (well logs and seismic
traces) depending of the geological features being sampled. Next section explores this idea in detail.

**Field data example**

The data set we used to test the applicability of Akay’s ideas to classify geophysical data consisted of two resistivity logs from two different wells (A and B), two sonic logs, two density logs, and a 3D seismic cube recorded with the aim of characterizing a clastic, Eocene reservoir in Lake Maracaibo, Venezuela, located at a depth of 12 300 feet. Wells A and B are located in areas that penetrate sandy and shally environments respectively.

**WBFA of resistivity logs**

We performed WBFA on the two resistivity logs. After doing a wavelet decomposition, we computed the variance of the coefficients at every label and plotted the result against its corresponding scale. Higher scales correspond to greater stretching in the analyzing wavelet and therefore, lower resolution. After testing the decomposition with Daubechies and biorthogonal wavelets, we selected biorthogonal wavelets since they provided better separation of the WBFA regression lines. Figure 2 shows the result. We observe distinctive separation between the slopes calculated from each log, which means this method (WBFA) has the potential of being used as the basis for facies identification in this area. Resistivity logs from wells with properties between these two end members can be classified with this method.

As we said before, increasing the scale of the wavelet transform implies in general more stretching in the analyzing wavelets and therefore, less resolution in the result. At such higher scales, the analyzing wavelets contain frequencies typical of the surface seismic frequency band. When we extend the WBFA of the resistivity logs to such higher scales, we observe a variation in the rate of change of the variance with scale at the fifth level, as Figure 3 shows.

This methodology can be used to relate the averaging processes of the subsurface properties between surface seismic and well log scale (which follow different power laws), to improve the estimation of reservoir properties based on the combination of seismic data and well logs, and to analyze large bandwidth data which may contain information
of unrelated averages of the subsurface properties (with different power laws), as Figure 3 shows.

FIG. 3. WBFA of the resistivity log using ten scales. Notice two different power laws in the behavior of the variance for high resolution (well logs) and low resolution (surfac seismic) scales.

WBFA of synthetic seismic traces

We checked whether the reflectivity series of each well responded differently to the WBFA depending on the dominant lithology of the target around each well. Figure 4 shows the result, which indicates it is still possible to recognize different lithologic characteristics using the reflectivity series in this area.

To generalize this result to surface seismic frequencies, we performed WBFA on synthetic seismic traces obtained after convolving the reflectivity series with a set of eight Ricker wavelets with central frequencies ranging from 20 to 256 Hz. We computed the slope and intercept of the line that best fitted the variance of the wavelets coefficients vs. scale for each synthetic trace. Figure 5 shows the results. For large frequencies, we obtain differences in slope and intercept between wells A and B. Differences in slope decrease as the central frequency decreases until lines become parallel at 20 Hz. Differences in intercept remain for all frequencies.

Results of Figure 5 indicate the methodology of classification of compressed surface seismic data based on slope and intercept that result after WBFA will be more robust as the frequency increases. For low frequencies, only differences in intercept will be significant.

When generating synthetic data, we also generated random, acoustic impedance logs (not shown). The results of the WBFA on such random logs did not exhibit the straight line behavior shown in Figures 2 and 4 that we obtained when analyzing the real resistivity and impedance logs. The behavior of the log-log plots for the random, synthetic well logs was erratic.
WBFA of 3D seismic data

To map the extension of different facies in the reservoir, we used the WBFA slope-intercept methodology to classify the 3D seismic data in a time window around the zone of interest. According to the results of Figure 5, we did not expect to see large differences in the slope of the lines after performing WBFA for traces around each well, since the frequency content of the 3D seismic data we used in this study varied only between 15 and 35 Hz. The result of the WBFA for the closest traces to wells A and B shown in Figure 6 confirmed this hypothesis: both lines are almost parallel and the separation is not as clear as when using well logs. However, when we performed the analysis for the 25 traces closest to each well, we obtained that slope and intercept of the different straight lines clustered in almost disjoint sets (Figure 7), which means the differences in the signals that WBFA reveals, even though subtle, are consistent and independent of random noise. This result is significant and suggests the WBFA slope-intercept scheme could be a valuable tool for facies identification using compressed seismic data.
FIG. 8. A facies classification map using WBFA.

Figure 8a shows a coherency slice (?) around the zone of interest. Notice the presence of a channel crossing the area. The result of classifying each trace of the 3D seismic data around the zone of interest using the WBFA slope-intercept methodology is shown in Figure 8b. The channel indicated in the coherency slice turns out to be, as expected, filled with sand. White areas in the map of Figure 8b could not be classified as either shale or sand.

Conclusions
We have presented a methodology, wavelet based fractal analysis (WBFA), to analyze the variance of the wavelet coefficients seismic traces at different scales. When applied to well logs, the method is able to discriminate properties of signals recored in areas with different shale and sand content. Even though the discriminating power of the method diminish for low frequency seismic data, useful results can still be obtained when applied to the problem of facies recognition using compressed data.

In this particular study, we found that two parameters that result from the WBFA, slope and intercept, were enough to classify the different facies. In other cases, slope and intercept may not be sufficient and we may need to use the variance of each scale of the wavelet transform without assuming a straight line model.

ACKNOWLEDGMENTS
We thank PDVSA-Intevep for permission to publish this work.